P	is the pressure;
$\Delta P_{02} = P_{02} - P_e$; $\Delta P = P - P_e$; $m = U_2/U_1$;	
ν	is the kinematic viscosity;
α	is the heat-transfer coefficient;
$\bar{\alpha}$	is the mean heat-transfer coefficient;
ρ	is the density.

Subscripts

1	parameters on the nozzle exit in the central part of the jet;
2	parameters of an annular jet;
m	axis;
e	external parameters;
o	stagnation parameters;
*	reverse flow to the center of the obstacle.

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EXTENT OF THE SUBSONIC DOMAIN IN A SUPERSONIC UNDEREXPANDED JET

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UDC 532.525:621.43.011

A method is proposed to determine the extent of the subsonic domain in a supersonic underexpanded jet. The method is based on representing the flow parameters in the form of series.

The qualitative flow pattern in a supersonic jet has been studied sufficiently well. It is known that a clear wave structure, conserved within the limits of several periods of the jet, is observed for $n < 5$. The flow parameters in such a jet are computed either by numerical methods [1] or by using approximate methods [2]. Computation of the stream parameters in the subsonic flow domain being formed during the nonregular reflection of a "hanging" compression shock from the jet axis (Fig. 1) is of definite difficulty. As computations have shown, the curvature of the contact surface being formed at the point C is sufficiently small. Hence, two flow schemes can be achieved which compare it to the flow of an inviscid gas in a channel with slightly cambered walls. In the first scheme, the contact surface forms a contracting channel, in whose minimal section the speed of sound is reached. This case corresponds to a negative slope of the velocity vector at the point C. In the second scheme the contact surface forms a channel of variable curvature in which the flow is first retarded somewhat and then accelerated to reach the speed of sound at the minimal section, as in the first case. This scheme corresponds to a positive angle θ_{6C}, characteristic for an underexpanded jet.

An approximate method is proposed for the analysis of the subsonic jet domain which is based on the assumption of one-dimensionality of the flow. We shall consider the flow parameters known up to the Mach

Leningrad Mechanics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 1, pp. 44-48, January, 1980. Original article submitted March 13, 1979.

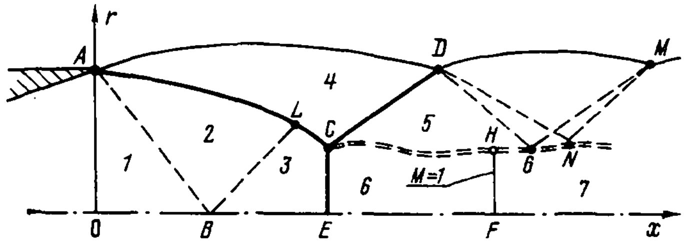

Fig. 1. Diagram of a supersonic underexpanded jet: ADM) jet boundary; AC, CE, CD) compression shocks; AB, BL, DG, DN, GM, NM) boundary characteristics; CHGN) contact surface; and 1, 2, 3, 4, 5, 6, 7) characteristic jet domains.

disk of the jet. We represent the pressure change in the subsonic domain in the form of the series

$$p_6(x) = p_{6E} + \left(\frac{dp}{dx}\right)_{6E} x + \left(\frac{d^2p}{dx^2}\right)_{6E} \frac{x^2}{2} + \ldots, \tag{1}$$

Here the subscript 6E refers to parameters at the point E from side of zone 6; the coordinate x is measured from the Mach disk. To determine the derivatives we use the continuity equation in coordinates coupled to the streamlines:

$$\frac{\partial\theta}{\partial n} + \frac{M^2-1}{\rho v^2}\frac{\partial p}{\partial s} + \frac{\sin\theta}{r} = 0. \tag{2}$$

The term $\sin\theta/r$ becomes indefinite on the jet axis. This indeterminacy is resolved by using a passage to the limit which yields the following result:

$$\lim_{\substack{\theta\to 0\\ r\to 0}} \frac{\sin\theta}{r} = \frac{\partial\theta}{\partial n}.$$

Therefore, on the jet axis

$$\frac{\partial\theta}{\partial n} = -\frac{M^2-1}{2\rho v^2}\frac{\partial p}{\partial s}.$$

The equation can be used in such a form in any continuous flow domain. In particular, we can write

$$\left(\frac{\partial\theta}{\partial n}\right)_{3E} + \frac{M_{3E}^2-1}{2\rho_{3E}v_{3E}^2}\left(\frac{\partial p}{\partial s}\right)_{3E} = \left(\frac{\partial\theta}{\partial n}\right)_{6E} + \frac{M_{6E}^2-1}{2\rho_{6E}v_{6E}^2}\left(\frac{\partial p}{\partial s}\right)_{6E}$$

for the point E from domains 3 and 6. The derivative $\partial\theta/\partial n$ characterizes the change in angle along lines orthogonal to the streamlines. Let us assume that such a line is the Mach disk generator, i.e., the jump CE in the line relative to the local velocity vector. Under such an assumption, which is verified for an underexpanded jet, $(\partial\theta/\partial n)_{3E} = (\partial\theta/\partial n)_{6E}$. Then the last equation is written in the form

$$\left(\frac{dp}{ds}\right)_{6E} = \frac{(M_{3E}^2-1)\,\rho_{6E}v_{6E}^2}{(M_{6E}^2-1)\,\rho_{3E}v_{3E}^2}\left(\frac{dp}{ds}\right)_{3E}. \tag{3}$$

Using the relationship for the normal shock and introducing the notation x in place of s for the axial streamline, we obtain an expression for the derivative from (3):

$$\frac{d\bar{p}_6}{dx} = -\left(\frac{2k}{k+1}M_3^2 - \frac{k-1}{k+1}\right)\left(\frac{2}{(k+1)M_3^2} + \frac{k-1}{k+1}\right)\frac{d\bar{p}_3}{dx}. \tag{4}$$

The pressure is here referred to the stagnation pressure at the nozzle exit; for simplicity in the writing, the subscript E is omitted.

Taking into account that $\bar{p}_3 = \pi(M_3)$ we have the derivative

$$\frac{d\bar{p}_3}{dx} = -kM_3\tau(M_3)\,\pi(M_3)\frac{dM_3}{dx}, \tag{5}$$

TABLE 1. Comparison between the Values of $\bar{p}_6$ Obtained by the Method Proposed and Those in [1]

Jet parameters				
	n	10	100	1000
	M_a	4	4	3
	k	1,3	1,4	1,3
	θ_a^0	10	10	10
$\bar{p}_6 \cdot 10^4$	Ref. [1]	10,5	1,53	3,5
	Eq. (8)	9,0	1,44	3,44

where dM_3/dx is determined during differentiating the relationship for M(x). For instance, we have from [2]

$$\frac{dM_3}{dx}=\left(\frac{M_3\sqrt{M_3^2-1}}{1+\frac{k-1}{2}M_3^2}-\frac{M_3}{\sqrt{M_3^2-1}}\right)^{-1}. \tag{6}$$

To determine the derivative $d^2\bar{p}_6/dx^2$ we differentiate (2). After some manipulation, we obtain

$$\frac{d^2\bar{p}_6}{dx^2}=\frac{k\bar{p}_6M_6}{M_6^2-1}\left\{\frac{M_3^2-1}{k\bar{p}_3M_3^2}\left(\frac{d^2\bar{p}_3}{dx^2}\right)+\frac{d\bar{p}_3}{dx}\left[\frac{2}{k\bar{p}_3M_3^3}\left(\frac{dM_3}{dx}\right)-\right.\right.$$
$$\left.\left.-\frac{M_3^2-1}{k\bar{p}_3^2M_3^2}\left(\frac{d\bar{p}_3}{dx}\right)\right]-\left(\frac{d\bar{p}_6}{dx}\right)\left[\frac{2}{k\bar{p}_6M_6^3}\left(\frac{dM_6}{dx}\right)-\frac{M_6^2-1}{k\bar{p}_6^2M_6^2}\left(\frac{d\bar{p}_6}{dx}\right)\right]\right\}. \tag{7}$$

The expressions for the derivatives $d^2\bar{p}_3/dx^2$ and d^2M_3/dx^2 are obtained as a result of differentiating the dependences (5) and (6).

Therefore, by using the relationships obtained, the two derivatives which are the coefficients in the series (1) can be evaluated. Calculation of the higher order derivatives results in quite awkward expressions. We limit ourselves to three terms of the series for an approximate analysis of the domain 6:

$$\bar{p}_6(x)=\bar{p}_{6E}+\left(\frac{d\bar{p}}{dx}\right)_{6E}x+\left(\frac{d^2\bar{p}}{dx^2}\right)_{6E}\frac{x^2}{2}. \tag{8}$$

The possibility of such a representation of the dependence $\bar{p}_6(x)$ is justified by comparing the values of the quantity $\bar{p}_6$ found from (8) with those obtained in [1]. The values of $\bar{p}_6$ presented in Table 1 have been obtained for a section at a distance $3r_a$ from the central shock, i.e., approximately at the site where the speed of sound is achieved.

Taking into account that $\bar{p}_6 = \bar{p}_{6*}$ for $M_6=1$, the location of the sonic section relative to the point E is determined from (8):

$$x^*=\frac{-\left(\frac{d\bar{p}}{dx}\right)_{6E}+\sqrt{\left(\frac{d\bar{p}}{dx}\right)_{6E}^2-2(\bar{p}_{6E}-\bar{p}_{6*})\left(\frac{d^2\bar{p}}{dx^2}\right)_{6E}}}{\left(\frac{d^2\bar{p}}{dx^2}\right)_{6E}}. \tag{9}$$

By knowing the pressure change, we can determine the distribution of the number M, while the shape of the slip line is found from the condition of constancy of the discharge in the subsonic domain:

$$q(M_6)r_6^2=q(M_{6c})r_c^2, \tag{10}$$

where $M_{6C} \approx M_{6E}$.

The method proposed to determine the extent of the subsonic zone of a jet does not take account of the formation of a mixing layer along the contact surface. The possibility of such an approach is justified by the comparison between the computed and experimental values of the quantity x^*. A graph of the dependence of x^* on the initial jet parameters $k = 1.4$ and $\theta_a = 10°$ is presented in Fig. 2. As follows from the graph, the quantity x^* increases as the Mach number M_a and the off-design of the exhaust grow. Available experimental data obtained by G. G. Shabalin (Leningrad Mechanics Institute) agree satisfactorily with the analysis: the greatest discrepancy between the values does not exceed 18%.

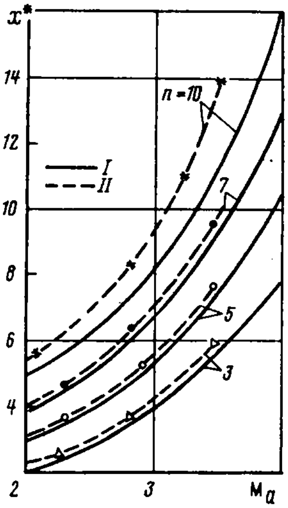

Fig. 2. Dependence of the extent of the subsonic domain x* on the initial jet parameters k = 1.4, θ_a = 10°: I) calculated; II) experiment.

NOTATION

k	is the ratio of the specific heats;
M	is the Mach number;
n	is the degree of exhaust off-design, orthogonal in direction to the streamline;
p	is the pressure;
r	is the radial coordinate;
s	is the coordinate along the streamline;
v	is the velocity;
x	is the longitudinal coordinate;
θ	is the slope of the velocity vector to the jet axis;
ρ	is the density;
$\pi(M) = p/p_0$, $\tau(M) = T/T_0$, $q(M) = F_*/F$	are the gas dynamic functions.

Subscripts

a nozzle exit;
o stagnation parameters;
* critical parameters; the other letter and number subscripts denote characteristic points and domains of the jet. Linear dimensions are referred to the nozzle radius at the exit.

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